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High-Throughput Characterization of Transition Metal Dichalcogenide Alloys: Thermodynamic Stability and Electronic **Band Alignment**

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cluster expansions, we compute the phase diagrams of 72 TMD monolayer °°° alloys and classify the mixing behavior. We show that ordered phases in general are absent at room temperature but that there exist some alloys that)°၀ have a stable Janus phase at room temperature. Furthermore, for a subset of these alloys, we quantify the band edge bowing and show that the band edge

positions for the mixing alloys can be continuously tuned in the range set by the boundary phases.

1. INTRODUCTION

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Monolayer transition metal dichalcogenides (TMDs) constitute a class of two-dimensional (2D) materials, commonly of MX₂ stoichiometry, where M is a transition metal (e.g., Mo, W, Zr, Hf, Ti, Pd, and Pt) and X is a chalcogen (S, Se, and Te). These monolayer compounds exhibit a structure where the transition metal is sandwiched between chalcogen atoms and exhibit hexagonal or trigonal symmetry (Figure 1a). TMDs have received a considerable amount of attention over the last decade, for example, due to the excellent optical properties of the semiconducting group VI TMD MoS_{2} , the prospects of vertical integration into heterostructures,² the emergent properties of moiré structures,^{3,4} and the outlook of using TMDs as catalysts for the hydrogen evolution reaction.⁵ Many TMD monolayers exhibit moderate band gaps of around 1-3 eV, which is suitable for photochemical applications or for use as channel materials in nanoelectronics.⁶⁻⁸ Due to the emergence of high- κ dielectrics as gate material in transistors such as ZrO_2 and HfO_2 , (2D) TMDs based on the same transition metals may also play a role in future high- κ transistor designs."

The electrical transport properties as well as the optical response are ultimately dependent on band edge positions. The position of the band edges are particularly important in devices based on heterojunctions. Specifically, TMD heterostructures with type-II alignment can be used for photoseparation of charge carriers.¹⁰ There are various ways to tailor the electronic properties, including functionalization,¹¹ Cou-lomb engineering,^{12–14} strain engineering,^{15,16} and alloying.^{14,17,18} While alloying may allow for a continuous tuning of the band edge position, not all alloys can be manufactured

due to their inherent thermodynamical properties. The main objectives of this work are therefore to map out the phase diagrams of a large number of binary TMD alloys and the band edge variations of promising alloy combinations.

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All of the binary alloys based on (Mo,W) $(S,Se)_2$ have been synthesized,¹⁹ usually in the hexagonal H structure (space group P6m2, ITCA number 187; Figure 1b), and have been thoroughly studied in the context of band gap engineering by composition control.^{18,20–25} It has been shown that these alloys are random at ambient conditions due to (very) small mixing energies.¹⁸ Although ordered phases have been found in computational studies,¹⁹ these are not observed in experiments,^{26,27} most likely due to the very small energy gains on the order of 1 meV compared to that of the respective disordered solution. In addition, the $MoS_{2x}Te_{2(1-x)}$ alloy has been fabricated.²⁸ For the optical properties, in particular, it is relevant whether alloying preserves the valley degree of freedom found in the boundary phases, that is, the nonalloyed systems. In this regard, it is noteworthy that, for example, $Mo_x W_{1-x}S_2$ has been recently shown to be robust in this regard.²

While the group VI TMD alloys have been extensively studied, the prospect of alloying these TMDs with transition metals from other groups of the periodic table has not been

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Figure 1. Overview of the 72 TMD monolayer alloys considered in this work in (a) space group $P\overline{3}m1$ (ITCA number 164) and (b) space group $P\overline{6}m2$ (ITCA number 187). The alloys consisted of either (c,d) two species on the metal (M) site and one (no alloying) on the chalcogen (X) site or (e,f) one species (no alloying) on the M site and two species on the X site. For space group 164, we considered Ti, Zr, Hf, Pd, and Pt on the M site, but we never mixed Ti/Zr/Hf with Pd/Pt. For space group 187, we considered Ti, Zr, Hf, Mo, and W. The X site was always occupied by S, Se, or Te, either just one of them (M-site mixing) or a mixture of two of them (X-site mixing).



Figure 2. Schematic phase diagrams of systems that show (a) phase separation (category A), (b) intermediate phase at 50% with a long-range inplane order (category B) and (c) intermediate phase at 50% with a long-range out-of-plane (or Janus-type) order (category C). (d–f) The critical temperatures (dotted, green lines) are identified as the position of the peak in heat capacity (blue lines), as calculated with MC simulations at 50% concentration. The peaks correspond to a rapid decline in the relevant order parameter (orange lines), here chosen as the (d) first-nearest neighbor short-range order, (e) in-plane structure factor, and (f) difference in concentration between the two layers (for details, see Supporting Information Note S3). Insets show snapshots of small parts of the structures in the MC simulations [top view in (d,e) and side view in (f)].

systematically addressed, although doping of, for example, MoS_2 with Ti has been reported.²⁹ In addition, there are various families of TMDs, including different structural prototypes, for which the alloying behavior is largely unexplored. The TMD monolayers based on Zr, Hf, Pd, and Pt preferentially crystallize in the trigonal T phase (space group $P\overline{3}m1$, ITCA number 164; Figure 1a)^{30–33} and exhibit indirect band gaps. The basic electronic structure and excitation spectra of Zr- and Hf-based boundary phase TMDs have been studied previously.³⁴ Some theoretical³⁵ and experimental^{36,37} studies on the electronic properties of the layered bulk alloys exist, but to the best of our knowledge,

the thermodynamic and electronic properties of T-TMD monolayer alloys have not been systematically addressed.

A special subclass of monolayer alloys are Janus monolayers, that is, ordered MXX' compounds where the chalcogens X and X' occupy the top and bottom layer sides of the TMD sheet. Janus monolayers have been suggested to be useful for band alignment engineering¹⁴ and as catalysts for the hydrogen evolution reaction.³⁸ Furthermore, theoretical studies have suggested that several materials may exhibit Janus phases.³⁹ While some Janus structures have been manufactured,⁴⁰ little is known about the general thermodynamical stability of these phases.

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Figure 3. Mixing behavior and critical temperatures for all 72 systems studied here. Each box indicates the predicted mixing behavior close to zero temperature: category A for non-mixing systems that phase separate (lateral heterostructures), category B for systems that form alloys with longrange order, and category C for systems that form Janus monolayers. Temperatures indicate the predicted critical temperatures (rounded to nearest 10 K) at which this quality is lost at 50% concentration. Colors are proportional to the critical temperatures.

In this work, we aim to provide a more comprehensive perspective of TMD monolayer alloys. We consider 72 binary alloys of the T and H structure types (Figure 1) and focus on their thermodynamic properties, specifically the underlying phase diagrams, and electronic properties, specifically band edge positions (for the semiconducting systems) and work functions (for the metallic systems). In the next section, we outline the methodology used in this work with details provided in the Supporting Information. This is followed by a description and discussion of the phase diagrams and mixing characteristics in Section 3.1. Models for the critical temperature and the structural categorization are presented in Section 3.2, while the computed band edge positions and bowing parameters are summarized in Section 3.3.

2. METHODOLOGY

Here, we study the thermodynamic and electronic properties of TMD alloys with chemical formula MX_2 (Figure 1) in the T structure (space group $P\overline{3}m1$, ITCA number 164) and the H structure (space group P6m2; ITCA number 187). For the T structure, we consider mixing on the M-site involving Ti, Zr, and Hf (nine systems) and Pd and Pt (three systems) as well as X-site mixing (15 systems). For the H structure, we consider mixing on the M-site, involving Ti, Zr, Hf, Mo, and W (30 systems), as well as X-site mixing (15 systems). These alloys were selected based on the known stability or metastability of the boundary phases, as documented in the Computational 2D Materials Database.³

To assess the thermodynamic properties of these TMD alloys, we constructed alloy cluster expansions^{41,42} (Supporting Information Note S2) using reference data from density functional theory (DFT) calculations^{41,43-49} (Supporting Information Note S1). These models were subsequently sampled using Monte Carlo (MC) simulations^{41,50} to obtain phase diagrams and critical temperatures (Supporting Information Note S3).

We then carried out a further characterization of the electronic properties of these systems, specifically focusing on the band gap and the position of the conduction and valence band edges using DFT calculations and special quasi-random

structures (SQSs) to mimic complete (fully random) mixing $^{41,43-47,51-55}$ (Supporting Information Note S4). From the latter analysis, we excluded alloy systems with very high critical temperatures, leaving us with 21 and 27 systems in structure types T and H, respectively.

3. RESULTS AND DISCUSSION

3.1. Thermodynamic Properties. The mixing behavior at zero temperature can be grouped into three different categories: (A) systems that exhibit two boundary phases separated by a miscibility gap and no long-range ordered mixed phase (Figure 2a) as well as systems that exhibit a long-range ordered phase at 50% composition with (B) in-plane order (Figure 2b) or (C) out-of-plane order (Figure 2c), a situation that is commonly referred as a Janus system.⁴⁰ The critical temperatures are evaluated by the peak in heat capacity, as calculated using MC simulations at 50% concentration. The peaks correspond to a rapid decline in the relevant order parameter, see Figure 2d-f.

Out of the 72 alloys considered, we find that 49 alloys fall into category A (Section 3.1.1), 12 into category B (Section 3.1.2), and 11 into category C (Section 3.1.3). These categories are discussed in more detail in the following sections.

3.1.1. Category A: Phase Separation into Boundary Phases. 3.1.1.1. M-Site Mixing with Elements from Different Groups. Among the 38 M-site alloys that are non-mixing at zero temperature, 18 have critical temperatures above approximately 1200 K (Figure 3a,b), which suggests that these combinations should be difficult to stabilize, if at all. This group comprises all alloys that mix species from different groups of the periodic table on the M-site such as Hf and Mo or W and Zr. According to the Hume-Rothery rules, wellestablished for the metallic alloys,⁵⁶ this behavior can be rationalized by the electronegativity differences. The group IV elements exhibit electronegativities of 1.3-1.5 on the Pauling scale,⁵⁷ whereas the group VI transition metals have values from 2.2 to 2.4 (Table S4). This indicates that the bonding in TMDs involving group IV species is much more ionic than in their counterparts from group VI. For some of these alloying

systems, for example, $Zr_xMo_{1-x^2}$ and $Ti_xMo_{1-x}Te_2$, there is even a sign difference in the Born effective charges (BECs) (Table S3), which is a further indication of the incompatibility of the mixing species. The critical temperature of alloys involving combinations of Zr or Hf with Mo or W is only weakly dependent on the specific transition metal combination. Rather, the critical temperature is primarily determined by the chalcogen species, with S- and Te-containing systems exhibiting the highest and lowest critical temperatures, respectively.

3.1.1.2. M-Site Mixing with Elements from the Same Group. The remaining 20 non-mixing M-site alloys have critical temperatures below approximately 900 K (Figure 3a,b). The nine systems that do not contain Ti even have critical temperatures below 200 K and should thus be readily miscible. The smaller miscibility in Ti-containing systems can be explained by the much larger difference in the lattice parameter between Ti- and Zr/Hf-based TMDs (Table S3), which is mirrored by the electronegativity differences (Table S4). In fact, as is discussed in Section 3.2, the lattice parameters of the boundary phases are in general strong predictors for miscibility. As in the case of M-site mixing with elements from different groups, the critical temperatures decrease monotonically from S via Se to Te.

3.1.1.3. X-Site Mixing. The critical temperatures for nonmixing X-site alloys are generally very low, with the sole exception of the $MS_{2x}Te_{2(1-x)}$ alloys with Pd and Pt that still exhibit values of 550 and 870 K, respectively (Figure 3c,d). Generally, the highest critical temperatures are obtained for S-Te alloys, which can be attributed to the large lattice mismatch between the MS_2 and MTe_2 boundary phases. We note that according to the Hume-Rothery rules, a 15% difference or more in the covalent radii should indicate immiscibility. Nevertheless, for the present systems, the covalent radius is not a good predictor for how well Te mixes with other chalcogen species. This is confirmed by the analysis in Section 3.2.

3.1.2. Category B: Systems with In-Plane Ordering. The systems in this category include both M-site and X-site mixing that all exhibit an intermediate phase at 50% with in-plane ordering (Figure 2b).

There are four M-site alloys, namely, $H-Mo_xW_{1-x}X_2$ with X = S, Se, and Te as well as $H-Hf_xZr_{1-x}Te_2$ (Figure 3b), and eight X-site alloys, including $H-MS_{2x}Se_{2(1-x)}$ with M = Mo, W, and Zr as well as five Hf-based systems (Figure 3c,d). The only X-site Hf-based alloy that does not exhibit this type of long-range order is $T-HfSe_{2x}Te_{2(1-x)}$.

Generally, the ordering tendency is weak and the critical temperatures are all below 200 K. For almost all practical purposes, these systems can be therefore considered as fully miscible.

3.1.3. Category C: Systems with Out-of-Plane (Janus-Type) Ordering. The systems in this category have the general composition MXX' where one chalcogen species (X) occupies the lattice sites above the transition metal and the other chalcogen species (X') occupies the sites below.

All Ti- and Zr-based X-site alloys in both T and H structure types with the single exception of H-ZrSSe have a Janus ground state at 50% composition. The critical (orderdisorder) temperature was obtained from MC simulations by following the difference in the composition in each individual layer. The thus obtained critical temperatures are above room temperature for T-TiSTe, T-TiSeTe, H-TiSTe, and H-TiSeTe with a value as high as 640 K for TiSTe, indicating remarkable stability. To the best of our knowledge, these Janus monolayers have not been synthesized yet. On the other hand, we find MoSSe, which has in fact been synthesized,⁴⁰ to be *thermodynamically* unstable in the present study (Figure S6). While the Janus monolayer is in fact the isomolar alloy with the highest energy, its energy is only about 13 meV/site equivalent to about 150 K, and thus, the driving force for decomposing such a structure is very small. This supports the notion that the growth conditions play a crucial role in enabling the formation of these structures.

It is instructive to consider the factors that lead to thermodynamically stable Janus monolayers by comparing the prototypical cases of H-MoSSe and T-ZrSSe. The basic parameters of the boundary systems in these two cases are very similar (Table S3). The most striking difference is the observed BECs, which differ qualitatively. While in H-MoSSe, the elements of the BEC tensor are negative, as has been shown for the boundary phases also in other studies,⁵⁸ in T-ZrSSe, these elements are positive. This finding is supported in Section 3.2, where we construct a classifier for phase behavior, which reveals that the BECs are important indicators for the appearance of thermodynamically favorable Janus structures.

3.1.4. Comparison with the Experiment. Experimentally, very little data are available with regard to the thermodynamic properties of TMD alloys, which can be attributed to the considerable difficulties associated with such measurements. It has been observed that phase separation in $MoS_{2x}Te_{2(1-x)}$ occurs mainly on the Te-rich side, ⁵⁹ which was suggested to be due to the formation of the distorted 1T' phase. In the present study, we have only considered isostructural phase diagrams, but nonetheless, we observe an asymmetric binodal (Figure S6). At 80% S, the alloy should phase-separate at around 275 K, whereas for 20%, this temperature increases to around 325 K. This suggests that the experimentally observed asymmetry could be already be explained by the thermodynamics of the isostructural system.

For the $Mo_{1-x}W_xS_2$ alloy, random mixing at room temperature for crystals grown with chemical vapor transport was recently shown,²⁷ which is what is expected from the present determination of the critical temperature for the order-disorder transition of 30 K.

The present study also has potential implications for the understanding of the formation and stability of 2D lateral heterostructures. It is apparent that the existence of a miscibility gap is not a prerequisite for forming lateral heterostructures with sharp interfaces. Such a configuration has, for example, been realized between MoS_2 and WS_2 ,^{60,61} for which both the present and earlier calculations predict a clear preference for mixing (Figure S5). The small negative mixing energy of down to -6.5 meV/site in combination with the mixing entropy should imply that such interfaces roughen and vanish at sufficient temperatures when sufficient kinetic activation enables the system to achieve a thermodynamically more favorable configuration.

On the other hand, for phase-separating systems, which can be synthesized in the mixed form under non-equilibrium growth conditions, high-temperature annealing may be used to construct temperature stable lateral heterostructures.⁶² In this context, Figures 3 and 6 provide guidance as to which heterostructures can and cannot be manufactured using annealing. For example, the proposed lateral heterostructure of T-ZrS₂ and T-HfS₂⁶³ is likely to require synthesis techniques beyond thermal annealing due to the low critical temperature of 30 K for the T- $Zr_xHf_{1-x}S_2$ alloy.

3.1.5. Comparison with Previous Modeling Studies. The critical temperatures of the $MS_{2x}Te_{2(1-x)}$ and $MSe_{2x}Te_{2(1-x)}$ alloys with M = Mo or W have been computed before in ref 20, which used a mean field approximation of the configurational entropy and the Perdew-Burke-Ernzerhof exchange-correlation functional. Here, we go beyond a mean field approximation by using MC simulations that explicitly sample the compositional configuration space and use a different exchange-correlation functional that has been shown to provide a good description of these systems.⁴⁸ Overall, we find a qualitative agreement, and in the case of the Se-Te alloys, there is a satisfying quantitative agreement as well. We predict that the critical temperature is 270 K (200 K) to be compared with 360 K (280 K) for the W (Mo)-based alloy. For the S-Te alloys, however, we find considerably lower critical temperatures of 370 K (240 K) to be compared with 690 K (490 K) for the W (Mo)-based alloy. If a mean field model for the entropy is used, the critical temperatures for the S-Te alloys increase by 70-80 K. It is also apparent that the present calculations yield a more asymmetric phase diagram, as already noted above. This suggests that the explicit treatment of the configurational entropy and the exchange-correlation are likely to play a role here.

It has been previously estimated based on a mean field approximation of the configurational entropy that the critical temperature for the order-disorder transition in the $Mo_{1-x}W_xS_2$ alloy for x = 1/3 and x = 2/3 is around 40 K,²³ which is very close to the value of 30 K obtained here.

3.2. Models for Predicting the Alloying Behavior. In the previous sections, we already indicated how certain properties of the boundary phases can serve as qualitative predictors for the properties of the mixed system. Now, we aim to analyze such relations more quantitatively. In the case of a metallic (binary) alloy, the Hume-Rothery rules are wellestablished as a set of basic conditions under which one can expect mixing. According to these (qualitative) rules, mixing ought to be possible if (i) the relative difference of the covalent radii of the components is less than 15% and the components have (ii) similar electronegativity as well as (iii) the same electronic valency. Even though these rules have been formulated for metallic systems, they can still partially explain the mixing behavior in several of the TMD alloys considered here. The Hume-Rothery conditions are, however, based on the properties of the free atoms and as such do not account for any many body interactions in the boundary phases. For example, in the case of MSTe alloys, the covalent radii of the chalcogens exhibit a 30% difference, yet the critical temperature ranges from around 200 K to around 1000 K, indicating that the contributions from interactions in the boundary phases are crucial for the alloying behavior.

It can be a daunting task to identify the features of the boundary phases that are the most suitable for predicting alloying ability. Thankfully, there are machine learning methods that are very well-suited for this task. For this purpose, here, we employ the sure independence screening and sparsifying operator (SISSO) approach that provides an automatized procedure for feature selection to construct predictors for the critical temperature as well as the alloy category (Figure 3).⁶⁴ To this end, we considered the following properties of the boundary phases: lattice parameters (a_0) , covalent radii, BECs, electronegativities, unit cell areas

(A), sheet modulus (B), ionization potential (IP), electron affinity (χ), and work function. Original features were constructed as the magnitude of the difference between the boundary phases. Since we found the sign of the BECs to be relevant in the case of the Janus monolayers (Section 3.1.3) for the categorization model, we constructed the signed BEC features as

$$\gamma_{\text{BEC}} = \frac{\operatorname{Tr}(\boldsymbol{\mathcal{Z}}_1^*) - \operatorname{Tr}(\boldsymbol{\mathcal{Z}}_0^*)}{\operatorname{Tr}(\boldsymbol{\mathcal{Z}}_0^*)}$$
(1)

where $\operatorname{Tr}(\mathbb{Z}_1^*) > \operatorname{Tr}(\mathbb{Z}_0^*)$ for the two boundary phases indicated with subscripts 0 and 1, respectively. This quantity provides the information regarding the BECs, removes the ambiguity due to the large anisotropy of the monolayers, and at the same time contains information regarding the sign of the BECs.

We note that some of the considered features are highly correlated, for example, the lattice parameter depends on the character of the bonding in the solid, which is also connected to the BECs, electronegativity, and bulk modulus.

With the exception of only three systems, the critical temperatures of the X-site alloys considered here are below 370 K. We therefore only consider M-site alloys here, for which we seek a linear relationship between the descriptor to be determined and the critical temperature. The descriptor that we find via the SISSO is

$$f = \sqrt{|\delta IP|} \sqrt[3]{|\delta B|} |\delta A|^2 \tag{2}$$

which yields a root mean square error (RMSE) of 292 K and a coefficient of determination R^2 of 0.956 (Figure 4). The



Figure 4. Parity plot of the critical temperature for M-site alloys as computed via MC simulations and the critical temperature model based on the descriptor in eq 2.

descriptor is not unique and depends on the features considered in the model as well as the set of alloys considered, as discussed in the original study ref 64. More crucially, it, however, provides clear indications as to which boundary phase features to consider when rationalizing the alloying behavior. Specifically, the descriptor here emphasizes that the ionization potential (or the valence band edge position) as well as the sheet modulus and unit cell area are important parameters. In addition, we constructed descriptors to enable a categorization of X-site alloys according to the classification introduced in Section 3.1. The resulting model has a 93% success rate in partitioning the different categories of mixing behavior (Figure 5) and uses two descriptors. The first



Figure 5. Classification of X-site alloys into ordered, Janus, or nonmixing ground states using the descriptors in eqs 3 and 4.

descriptor (abscissa in Figure 5) obtained via the SISSO is given by

$$d_1 = \gamma_{\rm BEC} - |\delta\chi| + \frac{\gamma_{\rm BEC}}{\gamma_{\rm BEC} + |\delta\chi|^3}$$
(3)

It is dimensionless and contains features directly related to bonding (γ_{BEC} and $|\delta\chi|$). The second descriptor (ordinate axis in Figure 5) is

$$d_2 = \frac{|\delta B|}{|\delta a_0|(1+\gamma_{BEC}^3)} \tag{4}$$

and has units of pressure. Unfortunately, neither of these two descriptors is amenable to a clear physical interpretation.

Two systems are incorrectly categorized. $HfS_{2x}Se_{2(1-x)}$ is categorized as both a Janus system and an ordered system, while in reality, it is an ordered system, yet the Janus structure is only $\leq 5 \text{ meV}$ f.u.⁻¹ compared to other configurations. $PdS_{2x}Se_{2(1-x)}$ (categorized as both non-mixing and Janus) on the other hand is a non-mixing system with a critical temperature of 80 K, but the Janus structure is 30 meV f.u.⁻¹ higher in energy than other configurations.

3.3. Electronic Structure. Following the analysis of the thermodynamic properties, we carried out a systematic evaluation of the valence and conduction positions as a function of the composition. From this analysis, we excluded systems with very high critical temperature, leaving us with 48 systems (Figure 6).

To first order, the band edge positions change linearly with composition between the boundary phases, a behavior that is often referred to as Vegard's law. The deviation from this dependence is commonly described by the bowing parameter b, which is defined via

$$\varepsilon(x) = (1-x)\varepsilon(x=0) + x\varepsilon(x=1) - bx(1-x)$$
(5)

where ε represents the concentration-dependent quantity in question. Note that according to this (standard) definition, -b/4 corresponds to the deviation from a linear interpolation at x = 0.5.

Band edge bowing parameters for $MX_{2x}X'_{2(1-x)}$ for M = Mo and W and X, X' = S, Se, and Te were already reported in ref 20. The current results are overall in good agreement with these data, albeit slightly smaller in magnitude (Table S5).

The general trend for X-site alloys is that the valence band bowing is negative and increasing in magnitude in the order $S_{2x}Se_{2(1-x)}$, $Se_{2x}Te_{2(1-x)}$, and $S_{2x}Te_{2(1-x)}$ (Figure 6c,d). Especially for the $S_{2x}Te_{2(1-x)}$ alloys, the magnitude of the



Figure 6. (a-d) Conduction band and valence band edge positions with respect to vacuum for H- and T-type M- and X-site alloys. Each box contains a heat map of the position of the conduction band (top) and valence band (bottom). The bowing parameters (see eq 5) in units of electron volts for the conduction band (valence band) are indicated in the top (bottom) of the box. (e,f) Conduction band minimum (CBM) and valence band maximum (VBM) as a function of the concentration in (e) T-ZrS_{2(1-x)}Se_{2x} and (f) H-TiSe_{2(1-x)}Te_{2x}. The bowing parameter is a measure of the deviation of the true band position from a linear interpolation of the band positions of the boundary phases, as indicated by the dotted, blue line in (f).

bowing parameter can exceed values of 1 eV. For the conduction band, the variations are generally smaller, and there is no apparent trend that extends through all TMDs. For example, for T-type Zr- and Hf-based TMDs, the bowing parameters for $S_{2x}Se_{2(1-x)}$, $Se_{2x}Te_{2(1-x)}$, and $S_{2x}Te_{2(1-x)}$ are all in the range between 0.07 and 0.09 eV. Furthermore, we find that for X-site alloys, the bowing parameter of the valence band is always negative and that for H-type X-site alloys, the conduction band bowing parameter is always negative. For T-type X-site alloys, the conduction band bowing parameter can have either sign. For example, in T-HfS_{2x}Se_{2(1-x}), the valence band exhibits a bowing parameter of -0.34 eV, while the conduction band bowing parameter is 0.09 eV.

For T-type M-site alloys, the magnitude of the bowing of the valence band edge is generally small, with the maximal (absolute) value of -0.05 eV occurring in T-Pd_xPt_{1-x}S₂ (Figure 6a,b). The conduction band edge exhibits significantly larger bowing in this class of alloys with values up to 1.10 eV (in T-Pd_xPt_{1-x}S₂). Finally, for H-type M-site alloys, the valence band bowing is larger in magnitude than for T-type M-site alloys with a maximal magnitude of -0.41 eV found in H-Ti_xZr_{1-x}S₂. The largest bowing of the conduction band for the H-type M-site alloys is found in Ti_xHf_{1-x}S₂ with a value of 0.29 eV.

The valence band and conduction band variations that are possible in the considered alloys are illustrated in Figure 6. Specifically, we find that the variation of the valence band can be very large in $MS_{2x}Te_{2(1-x)}$, for example, in $HfS_{2x}Te_{2(1-x)}$, the difference between the boundary phases is 1.7 eV. In general, the valence band position increases in the series S–Se–Te. The typical behavior is illustrated for T-ZrS_{2x}Se_{2(1-x)} (Figure 6e) and H-TiSe_{2x}Te_{2(1-x)} (Figure 6f).

It has been noted before that the magnitude of the valence band bowing parameter increases in the order $S_{2x}Se_{2(1-x)}$, $Se_{2x}Te_{2(1-x)}$, and $S_{2x}Te_{2(1-x)}$ for H-type Mo- and W-based Xsite alloys.²⁰ Our results show that this trend also holds true for Ti-, Hf-, and Zr-based alloys that exhibit H symmetry and Zr, Hf, Ti, Pd, and Pt alloys that exhibit T symmetry. The band edges exhibit in most cases moderate bowing with the exception of the X-site alloys $MS_{2x}Te_{2(1-x)}$. These are the Xsite mixed systems that exhibit the largest lattice constant mismatch and the largest differences in boundary phase features. Therefore, these alloys exhibit the largest variability of the considered alloys. The large bowing parameter in these compounds may however complicate band edge tuning since a small variation in composition is associated with a relatively large variation in the band edge position.

4. CONCLUSIONS

In this study, we have provided a comprehensive study of the phase diagrams and electronic structure of monolayer TMD alloys. It has been shown that mixing systems with in-plane order are absent at room temperature but that specifically, the Ti-based X-site alloys may exhibit a Janus ground state that remains the most stable structure far above room temperature. Furthermore, we have shown that M-site alloys with transition metals from different groups in the periodic table are associated with very high critical temperatures (>1000 K), due to which it is unlikely or at least very difficult for them to be manufactured. The band edges exhibit in most cases little bowing with the exception of X-site alloys that include both S and Te, implying that in many systems, the band edge

positions are relatively well-approximated by a linear interpolation of the values of the boundary phases.

ASSOCIATED CONTENT

Supporting Information

The Supporting Information is available free of charge at https://pubs.acs.org/doi/10.1021/acs.chemmater.2c01176.

Reference calculations for cluster expansion construction, cluster expansion construction, calculation of critical temperature and order parameters, electronic structure calculations, cluster expansion parameters, boundary phase features, elemental properties, bowing parameters, critical temperatures, valence band edge position of $HfS_{2x}Se_{2(1-x)}$, lattice constants as a function of concentration, mixing energies, and Mc sampling results (PDF)

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Notes

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High-throughput characterization of transition metal dichalcogenide alloys: Thermodynamic stability and electronic band alignment

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Supplementary References

Supplementary Notes

Supplementary Note S1: Reference calculations for cluster expansion construction

To generate reference data for the construction of alloy cluster expansions (CEs) (Supplementary Note S2), we carried out density functional theory (DFT) calculations using the projector augmented wave (PAW) method [1] as implemented in the Vienna ab-initio simulation package [2, 3]. The exchange-correlation contribution was represented using the van-der-Waals density functional method [4, 5] with consistent exchange (vdW-DF-cx) [6], which has been shown to be very well suited for transition metal dichalcogenides (TMDs) [7]. The plane-wave cutoff energy was set to 340 eV and we used the PBE PAW setups in version 54. The atomic positions and cell metric were relaxed until residual forces were less than 30 meV Å⁻¹ and residual stresses below approximately 2 kbar. Prior to the relaxation, a vacuum region of at least 28 Å was introduced along the direction of the surface normal to minimize interactions between periodic images, and it was asserted that after relaxation the layers were separated by at least 15 Å; in the vast majority of cases it amounted to at least 24 Å. During relaxations the Brillouin zone was sampled with a \vec{k} -point density of 0.25 Å⁻¹ and first-order Methfessel-Paxton smearing with a width of 0.1 eV. Final energy calculations were carried out for the relaxed structures with a \vec{k} -point density of 0.1 Å⁻¹ and the tetrahedron method with Blöchl corrections using a smearing width of 0.05 eV.

Structures were generated using the enumeration algorithm introduced in Ref. 8 as implemented in ICET [9]. Calculations were carried out for at least 27 structures (all structures with up to 15 atoms) and at least 28 structures (all structures up to 9 atoms) for M and X-site mixing, respectively. The sufficiency of this data set size was confirmed by testing the convergence for five systems with up to 100 structures, which showed an insignificant improvement in model accuracy.

The DFT data used for CE construction is provided in the form of ASE databases in the ZENODO data set associated with this publication [10].

Supplementary Note S2: Cluster expansion construction

In this work, CEs were constructed using the ICET package [9]. In the alloy CE approach, the mixing energy of an interacting many-body system can be expanded in cluster functions as

$$\Delta E_{\rm mix}(\sigma) = \sum_{\alpha} m_{\alpha} J_{\alpha} \langle \Gamma(\sigma) \rangle_{\alpha}, \tag{1}$$

where α is the index of an orbit (or representative cluster), m_{α} is the multiplicity of the orbit, Γ are cluster functions, $\vec{\sigma}$ denotes the occupation vector, and J_{α} are the so-called effective cluster interactions (ECIs) that are fitted to DFT data. As a result of the nearsightedness of atomic interactions, the magnitude of the ECIs quickly decay with distance and expansion order, which allows one to truncate the expansion. Here, we considered clusters with one, two, and three sites (singlets, pairs and triplets) in addition to the empty cluster (zerolet). For each alloy system, we then constructed series of CEs with an increasing number of pair and triplet terms. Each individual CE was fitted using the automatic relevance detection regression approach and the default hyperparameters as implemented in SCIKIT-LEARN 1.0 [11]. Among the set of CEs, we selected the one that minimized the Bayesian information criterion (BIC) score, while ensuring a high R^2 value for the validation set and a reasonable number of ECIs. In the majority of the systems, triplet clusters were not needed to achieve a satisfactory CE. Cutoffs, cross-validation scores, and other key quantities are summarized in Table S1 and Table S2. Generally the average root mean square errors over the validation sets are (i.e. the cross-validation scores) are very low and the coefficient of determination R^2 is very close to 1. The largest errors are obtained for systems with M-site mixing that combine species from the groups 4 (Ti, Zr, Hf) and 6 (Mo, W) of the period table, which also exhibit a very wide range of mixing energies and are very immiscible as indicated by their high critical temperatures. All CEs used in production are also included in the ZENODO data set associated with this publication [10].

Supplementary Note S3: Calculation of critical temperatures and order parameters

The alloy CEs were sampled via Monte Carlo (MC) simulations using the MCHAMMER module of the ICET package [9], in order to calculate critical temperatures. To this end, supercells with approximately 1,500 primitive cells were constructed and randomly populated with equal amounts of the two alloyants (50% concentration). These shape of the supercells were chosen in order to accommodate the ground state at 50% concentration. Simulations in the canonical ensemble were commenced at 5000 K and the temperature was lowered in steps of 10 K after 50 MC sweeps (1 MC sweep = N steps, where N is the number of atoms in the supercell), using the last configuration at one temperature as the starting configuration of the next. Thereafter, MC simulations were continued in parallel for a total of 3,000 MC sweeps at each temperature, and finally 30,000 MC sweeps were carried out at all temperatures within 250 K of the critical temperature. To allow for equilibration, the first 1000 MC sweeps were discarded before data collection.

Critical temperatures T_C were estimated as the temperature with maximum heat capacity C, calculated using the variance of the potential energy E according to $C = (\langle E^2 \rangle - \langle E \rangle^2) / k_B T^2$. We also confirmed that this peak coincided with a rapid decrease of the relevant order parameter, here chosen as the Warren–Cowley short-range order parameter [12] for nearest-neighbor pairs in non-mixing systems, long-range order calculated as structure factors with the relevant \mathbf{q} vector for ordering systems, and the difference in composition between upper and lower layer for Janus systems; all normalized to yield an order parameter of 1 in the perfectly ordered system.

Supplementary Note S4: Electronic structure calculations

We assessed the variation of band gaps and band edge positions with composition for 48 out of the 72 alloys considered in this work. To this end, we employed special quasi-random structures (SQSs) [13] to mimic complete (fully random) mixing that were generated using the algorithm described in Ref. 14 as implemented in ICET [9], yielding structures contained between 24 and 48 atoms. DFT calculations were again carried out using the PAW method [1] as implemented in the Vienna ab-initio simulation package [2, 3].

For structural relaxations, the exchange-correlation contribution was represented using the vdW-DF-cx method [4–6] as before (Supplementary Note S1). The plane-wave cutoff energy was set to 340 eV and we used the PBE PAW setups in version 54. The lattice constants were linearly interpolated between the boundary phases, which is motivated by the very small deviations from Vegard's rule that are apparent from the results for enumerated structures (Figure S2, Figure S3). The atomic positions were relaxed at fixed cell metric until residual forces were less than 30 meV Å⁻¹. Prior to the relaxation, a vacuum region of at least 28 Å was introduced along the direction of the surface normal to minimize interactions between periodic images, and it was asserted that after relaxation the layers were separated by at least 15 Å; in the vast majority of cases it amounted to at least 24 Å. During relaxations the Brillouin zone was sampled with a \vec{k} -point density of 0.25 Å⁻¹ and first-order Methfessel-Paxton smearing with a width of 0.1 eV.

We then calculated band gaps and band edge positions for all boundary phases as well as selected SQSs using the HSE06 hybrid exchange-correlation functional [15, 16], which is expected to provide a much improved description of band gaps and band edge positions. Comparison of the results from HSE06 and vdW-DF-cx calculations (Figure S1) shows that the difference can be well approximated by a rigid shift. Given the very high computational cost of the hybrid calculations, we therefore resorted to a simple interpolation procedure for the majority of the alloys. Specifically, the band edge variations were computed as

$$\varepsilon_x = \varepsilon_1^{\text{HSE06}} x + (1-x)\varepsilon_0^{\text{HSE06}} - b^{\text{vdW-DF-cx}}(1-x)x, \qquad (2)$$

where boundary phases band edges are indexed with 0 and 1.

Supplementary Tables

Table S1: Cluster expansion parameters for M-site mixing alloys. Cutoffs are given in units of the respective lattice parameter a_0 and indicate the longest allowed distance between any two atoms in the cluster. The cross-validation score is the average of ten root-mean square errors obtained by splitting the training set into 90% training structures and 10% validation structures, fitting with the former and calculating the root-mean square error of the latter. The R^2 score is the average R^2 score obtained for the validation structures in the same tenfold split of the training set.

Formula	Pair cutoff	Triplet cutoff	Cross-validation score	R^2	Nonzero ECIs		
	(<i>a</i> ₀)	(a_0)	(mev/formula unit)				
Trigonal (T)	Trigonal (T), $P\bar{3}m1$, ITCA number 164						
$(\mathrm{Ti},\mathrm{Zr})\mathrm{S}_2$	1.59	_	1.8	0.99	6		
$(Ti, Zr)Se_2$	1.62	_	1.9	0.99	5		
$(Ti, Zr)Te_2$	1.85	—	1.5	0.98	6		
$(Hf, Ti)S_2$	1.55	—	1.4	0.99	6		
$(Hf, Ti)Se_2$	1.65	—	1.4	0.99	5		
$(Hf, Ti)Te_2$	1.57	—	1.5	0.98	5		
$(Hf, Zr)S_2$	2.75	—	0.2	0.88	5		
$(Hf, Zr)Se_2$	1.87	—	0.5	0.40	4		
$(Hf, Zr)Te_2$	4.10	—	0.9	0.87	10		
$(\mathrm{Pd},\mathrm{Pt})\mathrm{S}_2$	1.07	—	0.3	0.52	4		
$(\mathrm{Pd},\mathrm{Pt})\mathrm{Se}_2$	0.79	—	0.2	0.95	2		
$(\mathrm{Pd},\mathrm{Pt})\mathrm{Te}_2$	0.73	—	0.3	0.98	3		
Hexagonal (I	H), $P\bar{6}m2$, ITC	CA number 187					
$(Ti, Zr)S_2$	1.75	_	1.6	0.99	4		
$(Ti, Zr)Se_2$	1.68	_	1.2	0.99	4		
$(Ti, Zr)Te_2$	1.57	_	0.9	0.99	5		
$(Hf, Ti)S_2$	1.75	—	1.1	0.99	5		
$(Hf, Ti)Se_2$	1.68	—	0.8	0.99	5		
$(Hf, Ti)Te_2$	1.57	—	0.7	0.99	5		
$(Mo, Ti)S_2$	1.28	0.92	14.4	0.95	4		
$(Mo, Ti)Se_2$	1.76	0.88	15.2	0.95	8		
$(Mo, Ti)Te_2$	2.86	—	17.8	0.88	6		
$(\mathrm{Ti}, \mathrm{W})\mathrm{S}_2$	0.88	—	14.1	0.95	3		
$(Ti, W)Se_2$	0.84	—	16.6	0.93	2		
$(Ti, W)Te_2$	1.18	—	18.1	0.89	5		
$(Hf, Zr)S_2$	1.64	—	0.1	0.99	5		
$(Hf, Zr)Se_2$	1.58	—	0.1	0.90	6		
$(Hf, Zr)Te_2$	1.87	—	0.2	0.70	7		
$(Mo, Zr)S_2$	3.18	—	15.3	0.97	6		
$(Mo, Zr)Se_2$	1.76	—	15.1	0.98	6		
$(Mo, Zr)Te_2$	2.86	2.00	10.5	0.97	7		
$(W, Zr)S_2$	1.83	—	17.0	0.97	6		
$(W, Zr)Se_2$	1.59	—	20.0	0.96	6		
$(W, Zr)Te_2$	1.65	—	15.3	0.95	5		
$(\mathrm{Hf},\mathrm{Mo})\mathrm{S}_2$	1.83	—	17.2	0.97	6		
$(Hf, Mo)Se_2$	1.76	—	18.6	0.96	6		
$(Hf, Mo)Te_2$	2.86	_	15.7	0.95	5		
$(Hf, W)S_2$	1.83	_	17.8	0.97	6		
$(Hf, W)Se_2$	1.59	_	19.3	0.96	6		
$(Hf, W)'Te_2$	1.65	_	16.2	0.95	5		
$(Mo, W)S_2$	2.38	_	0.1	1.00	4		
$(Mo, W)Se_2$	2.29	_	0.1	1.00	4		
(Mo, W)Te ₂	2.14	1.43	0.0	1.00	7		

Table S2: Cluster expansion parameters for X-site mixing alloys. Cutoffs are given in units of the respective lattice parameter a_0 and indicate the longest allowed distance between two atoms in the cluster. The cross-validation score is the average of ten root-mean square errors obtained by splitting the training set into 90% training structures and 10% validation structures, fitting with the former and calculating the root-mean square error of the latter. The R^2 score is the average R^2 score obtained for the validation structures in the same tenfold split of the training set.

Formula	Pair cutoff (a_0)	Triplet cutoff (a_0)	Cross-validation score (meV/formula unit)	\mathbb{R}^2	Nonzero ECIs	
Trigonal (T), $P\bar{3}m1$, ITCA number 164						
$Ti(S, Se)_2$	1.12	_	0.7	0.98	8	
$Ti(S, Te)_2$	1.29	0.86	6.7	0.92	11	
$Ti(Se, Te)_2$	1.07	_	2.3	0.95	7	
$\operatorname{Zr}(S, \operatorname{Se})_2$	1.19	_	0.5	0.96	6	
$\operatorname{Zr}(S, \operatorname{Te})_2$	1.23	—	5.3	0.83	8	
$\operatorname{Zr}(\operatorname{Se},\operatorname{Te})_2$	1.31	1.08	1.0	0.98	13	
$\mathrm{Hf}(\mathrm{S},\mathrm{Se})_2$	2.29	_	0.2	0.99	9	
$\mathrm{Hf}(\mathrm{S},\mathrm{Te})_2$	2.15	_	3.4	0.91	6	
$\mathrm{Hf}(\mathrm{Se},\mathrm{Te})_2$	2.22	1.35	1.4	0.96	10	
$Pd(S, Se)_2$	1.86	_	1.0	0.98	7	
$Pd(S, Te)_2$	1.91	_	7.2	0.96	5	
$Pd(Se, Te)_2$	1.91	_	2.7	0.95	6	
$Pt(S, Se)_2$	2.13	_	1.1	0.99	7	
$Pt(S, Te)_2$	2.16	_	6.5	0.98	6	
$Pt(Se, Te)_2$	1.76	—	1.9	0.98	6	
Hexagonal (I	H), $P\bar{6}m2$, ITC	CA number 187				
$\mathrm{Ti}(\mathrm{S},\mathrm{Se})_2$	2.12	_	1.0	0.99	8	
$\mathrm{Ti}(\mathrm{S},\mathrm{Te})_2$	2.27	_	17.8	0.84	8	
$\mathrm{Ti}(\mathrm{Se},\mathrm{Te})_2$	1.16	_	2.3	0.98	9	
$\operatorname{Zr}(S, \operatorname{Se})_2$	2.05	—	1.8	0.94	7	
$\operatorname{Zr}(S, \operatorname{Te})_2$	1.14	—	8.7	0.94	6	
$\operatorname{Zr}(\operatorname{Se},\operatorname{Te})_2$	1.18	—	2.0	0.98	8	
$\mathrm{Hf}(\mathrm{S},\mathrm{Se})_2$	1.24	—	0.9	0.98	6	
$\mathrm{Hf}(\mathrm{S},\mathrm{Te})_2$	2.15	_	10.1	0.90	7	
$\mathrm{Hf}(\mathrm{Se},\mathrm{Te})_2$	2.07	_	1.2	0.99	8	
$Mo(S, Se)_2$	2.38	1.59	0.3	1.00	11	
$Mo(S, Te)_2$	2.29	1.53	3.1	0.99	11	
$Mo(Se, Te)_2$	2.29	1.53	0.5	1.00	11	
$W(S, Se)_2$	1.37	0.92	0.3	1.00	11	
$W(S, Te)_2$	1.32	0.88	3.4	0.99	11	
$W(Se, Te)_2$	1.32	0.88	1.3	1.00	11	

Table S3: Boundary phase features. Data used in the construction of models for the critical temperatures and categorization of X-site alloys. The band edges (VBM and CBM) are computed on the level of conventional DFT. The bulk modulus is computed as the 3D counterpart i.e. $B = (c_{11} + 2c_{12})/3$ and then multiplied by the out-of-plane lattice vector.

	a_0 (Å)	VBM (eV)	CBM (eV)	$B \;(\text{eV}\text{\AA}^{-2})$	$\operatorname{Tr}(\mathcal{Z}_{\mathrm{M}}^{*})$ (e)	$\operatorname{Tr}(\mathcal{Z}_X^*)$ (e)
Trigonal (T), $P\bar{3}m1$, ITCA number 164						
HfS ₂	3.60	-6.4	-5.2	250	14.1	-7.0
$HfSe_2$	3.72	-5.5	-5.1	216	15.2	-7.6
HfTe_2	3.91	-4.8	-4.7	152	16.9	-8.4
MoS_2	3.12	-5.1	-5.1	194	-12.2	6.3
$MoSe_2$	3.23	-4.7	-4.6	240	-10.3	5.2
$MoTe_2$	3.45	-4.4	-4.3	208	-9.6	4.8
PdS_2	3.50	-6.4	-5.1	248	5.7	-2.9
$PdSe_2$	3.67	-5.5	-4.8	200	4.9	-2.5
$PdTe_2$	3.94	-4.6	-4.5	165	3.3	-1.6
PtS_2	3.53	-6.4	-4.6	289	5.0	-2.5
$PtSe_2$	3.70	-5.8	-4.4	253	4.2	-2.1
$PtTe_2$	3.96	-4.8	-4.1	239	2.3	-1.1
TiS_2	3.36	-5.8	-5.8	243	14.6	-7.3
$\tilde{\mathrm{TiSe}}_2$	3.49	-5.4	-5.3	197	18.8	-9.4
TiTe_2	3.68	-4.9	-4.8	168	9.2	-4.6
WS_2	3.14	-5.0	-4.9	168	-16.3	8.2
WSe_2	3.24	-4.5	-4.4	248	-10.8	5.4
WTe_2	3.46	-4.1	-4.0	193	-10.6	5.3
ZrS_2	3.64	-6.5	-5.4	233	14.5	-7.3
$ZrSe_2$	3.75	-5.6	-5.2	202	16.0	-8.0
ZrTe_2	3.90	-5.0	-4.9	126	15.3	-7.7
Hexagor	nal (H), P	$\bar{6}m2$, ITCA i	number 187			
HfS_2	3.50	-7.0	-5.9	348	9.6	-4.8
$HfSe_2$	3.62	-6.4	-5.6	295	10.0	-5.0
$HfTe_2$	3.85	-5.5	-5.3	216	12.8	-6.4
MoS_2	3.15	-6.0	-4.2	432	-2.4	1.2
$MoSe_2$	3.28	-5.4	-3.9	358	-3.9	2.0
$MoTe_2$	3.50	-5.0	-3.8	284	-6.9	3.4
TiS_2	3.30	-6.9	-6.1	321	9.5	-4.8
TiSe_2	3.45	-6.2	-5.7	279	10.2	-5.1
TiTe_2	3.68	-5.3	-5.3	196	19.5	-9.7
WS_2	3.15	-5.8	-3.9	471	-1.2	0.6
WSe_2	3.28	-5.2	-3.5	368	-2.7	1.3
WTe_2	3.50	-4.8	-3.6	278	-5.5	2.8
ZrS_2	3.53	-6.9	-6.0	321	9.7	-4.9
ZrSe_2	3.66	-6.4	-5.6	275	10.0	-5.0
ZrTe_2	3.86	-5.6	-5.3	205	13.6	-6.8

Table S4: Elemental properties. Elemental properties of considered elements. The electronegativity (χ) scale is the Pauling scale and the covalent radius (r_c) is in units of pm.

	χ	$r_c (\rm pm)$
Mo	2.16	154
W	2.36	162
Zr	1.33	175
$\mathbf{H}\mathbf{f}$	1.30	187
Ti	1.54	160
Pd	2.20	139
Pt	2.28	136
Te	2.10	138
Se	2.55	120
S	2.58	105

Table S5: Bowing parameters. Bowing parameters for the $MX_{2x}X'_{2(1-x)}$ with M=Mo, W and X, X'=S, Se, Te alloys. Values in parenthesis are from Ref. 17.

	$b_{\rm VBM}~({\rm eV})$	$b_{\rm CBM}~({\rm eV})$
$MoS_{2x}Se_{2(1-x)}$	-0.07(-0.22)	-0.23(-0.18)
$MoSe_{2x}Te_{2(1-x)}$	-0.22 (-0.36)	-0.11 (-0.24)
$MoS_{2x}Te_{2(1-x)}$	-0.98(-1.17)	-0.55 (-0.76)
$WS_{2x}Se_{2(1-x)}$	0.00 (-0.21)	-0.16(-0.16)
$WSe_{2x}Te_{2(1-x)}$	-0.13(-0.32)	-0.07 (-0.20)
$WS_{2x}Te_{2(1-x)}$	-0.84(-1.06)	-0.46(-0.70)

System	Symmetry	Critical temperature [K]	System	Symmetry	Critical temperature [K]
$(Hf, Zr)S_2$	Т	30	$Zr(S, Se)_2$	Т	20
$\operatorname{Zr}(S, \operatorname{Se})_2$	Т	20	$(Hf, Ti)Se_2$	Η	450
$\operatorname{Zr}(S, \operatorname{Se})_2$	Η	10	$(W, Zr)Te_2$	Η	2460
$(Mo, Zr)S_2$	Η	4230	$(Pd, Se)Te_2$	Т	190
$W(S, Te)_2$	Η	370	$(Ti, Zr)S_2$	Т	910
$(Hf, Mo)Se_2$	Η	3360	$(Mo, W)S_2$	Η	30
$Pd(S, Te)_2$	Т	550	$(Pd, Pt)S_2$	Т	10
$Pt(Se, Te)_2$	Т	280	$Mo(S, Se)_2$	Η	30
$(Hf, Zr)Te_2$	Т	50	$Hf(S, Se)_2$	Т	20
$W(S, Se)_2$	Η	30	$(Pd, Pt)Te_2$	Т	60
$(Hf, Zr)Se_2$	Т	20	$Hf(S, Te)_2$	Т	120
$Zr(S, Te)_2$	Т	240	$Ti(S, Te)_2$	Т	640
$(Mo, Ti)S_2$	Η	2090	$(W, Zr)Se_2$	Η	3410
$\mathrm{Ti}(\mathrm{S},\mathrm{Te})_2$	Η	340	$(Ti, Zr)Se_2$	Η	570
$(Hf, Zr)Se_2$	Η	10	$(Ti, Zr)S_2$	Η	710
$(Hf, Ti)S_2$	Т	730	$(Mo, W)Se_2$	Η	10
$Zr(S, Te)_2$	Η	20	$Pd(S, Se)_2$	Т	80
$(Ti, W)Te_2$	Η	1190	$(Hf, W)S_2$	Η	3950
$(Hf, Ti)Te_2$	Т	450	$(Hf, W)Se_2$	Η	3240
$Pt(S, Se)_2$	Т	120	$W(Se, Te)_2$	Η	270
$Hf(Se, Te)_2$	Т	110	$(Hf, Mo)S_2$	Η	4080
$(W, Zr)S_2$	Η	4140	$Mo(Se, Te)_2$	Η	200
$(Hf, Ti)S_2$	Η	550	$Zr(Se, Te)_2$	Т	170
$(Ti, Zr)Te_2$	Т	520	$Mo(S, Te)_2$	Η	240
$(Ti, Zr)Te_2$	Η	390	$Ti(Se, Te)_2$	Т	350
$(Mo, Zr)Se_2$	Η	3630	$(Hf, Mo)Te_2$	Η	2350
$Pt(S, Te)_2$	Т	870	$(Hf, Zr)S_2$	Η	20
$(Hf, Ti)Se_2$	Т	730	$Hf(S, Se)_2$	Т	90
$(\mathrm{Pd},\mathrm{Pt})\mathrm{Se}_2$	Т	30	$Hf(Se, Te)_2$	Η	10
$(Hf, Ti)Te_2$	Η	300	$Ti(S, Se)_2$	Η	60
$Hf(S, Te)_2$	Η	10	$(Hf, Zr)Te_2$	Η	10
$(Mo, Zr)Te_2$	Η	2410	$(Ti, W)Se_2$	Η	1940
$(Hf, W)Te_2$	Η	2430	$(Mo, Ti)Se_2$	Η	1890
$\mathrm{Ti}(\mathrm{Se},\mathrm{Te})_2$	Η	370	$(Mo, W)Te_2$	Η	10
$Zr(Se, Te)_2$	Н	190	$Ti(S, Se)_2$	Т	150
$(Mo, Ti)Te_2$	Η	1350	$(Ti, Zr)Se_2$	Т	880

 Table S6: Critical temperatures. Critical temperatures for all considered alloys.

Supplementary Figures



Figure S1: Valence band edge position for $HfS_{2x}Se_{2(1-x)}$. Comparison between HSE06 and the band edges from HSE06 with the bowing parameter from vdW-df-cx.



Figure S2: Lattice constant as a function of concentration for Mo and W-based alloys (spacegroup 187). top: M-site mixing; bottom: X-site mixing



Figure S3: Lattice constant as a function of concentration for Hf, Zr and Pd-based alloys (spacegroup 164). top: M-site mixing; bottom: X-site mixing



Figure S4: Mixing energies (part 1) for Hf/Mo/Ti/W/Zr-based M-site mixing (spacegroup 187). Small gray circles mark mixing energies of enumerated structures obtained using the CE.



Figure S5: Mixing energies (part 2) for Hf/Mo/Ti/W/Zr-based M-site mixing (spacegroup 187). Small gray circles mark mixing energies of enumerated structures obtained using the CE.



Figure S6: Mixing energies for Hf/Mo/Ti/W/Zr-based X-site mixing (spacegroup 187). Small gray circles mark mixing energies of enumerated structures obtained using the CE.



Figure S7: Mixing energies for Hf/Ti/Zr-based M-site mixing (spacegroup 164). Small gray circles mark mixing energies of enumerated structures obtained using the CE.



Figure S8: Mixing energies for Hf/Ti/Zr-based X-site mixing (spacegroup 164). Small gray circles mark mixing energies of enumerated structures obtained using the CE.



Figure S9: Mixing energies for Pd/Pt-based M-site mixing (spacegroup 164). Small gray circles mark mixing energies of enumerated structures obtained using the CE.



Figure S10: Mixing energies for Pd/Pt-based X-site mixing (spacegroup 164). Small gray circles mark mixing energies of enumerated structures obtained using the CE.



Figure S11: Results from MC sampling for Mo/W-based M-site mixing (spacegroup 187). XXX.



Figure S12: Results from MC sampling for Mo/Ti-based M-site mixing (spacegroup 187). XXX.



Figure S13: Results from MC sampling for Mo-based X-site mixing (spacegroup 187). XXX.



Figure S14: Results from MC sampling for W-based X-site mixing (spacegroup 187). XXX.



Figure S15: Results from MC sampling for Hf/Zr-based M-site mixing (spacegroup 164). XXX.



Figure S16: Results from MC sampling for Hf/Ti-based M-site mixing (spacegroup 164). XXX.



Figure S17: Results from MC sampling for Ti/Zr-based M-site mixing (spacegroup 164). XXX.



Figure S18: Results from MC sampling for Hf-based X-site mixing (spacegroup 164). XXX.



Figure S19: Results from MC sampling for Zr-based X-site mixing (spacegroup 164). XXX.



Figure S20: Results from MC sampling for Pd/Pt-based M-site mixing (spacegroup 164). XXX.



Figure S21: Results from MC sampling for Pd-based X-site mixing (spacegroup 164). XXX.



Figure S22: Results from MC sampling for Pt-based X-site mixing (spacegroup 164). XXX.

Supplementary References

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